

SIGNAL SPACE

Framework to
understand Digital
Communication
Systems.

Consider two pulses.
 $P_1(t), P_2(t)$

$$\int_{-\infty}^{\infty} p_1^2(t) dt = \int_{-\infty}^{\infty} p_2^2(t) dt$$
$$= \mathcal{E}_p = 1$$

Both pulses have unit Energy.

Further, more importantly,

$$\int_{-\infty}^{\infty} p_1(t) p_2(t) dt = 0$$

Both pulses are orthogonal.

$P_1(t), P_2(t)$

Constitute the orthonormal
basis for a signal space

ORTHOGONAL + NORMAL

Both pulses
are orthogonal.

Pulses are
normalized to
unit Energy

SIGNAL SPACE

Linear combination
of orthonormal basis
signals.

Signal Space $\alpha_1 p_1(t) + \alpha_2 p_2(t) \in S$

Real numbers

All such linear combinations are elements of signal space.

Ex: $P_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_1 t)$

$$0 \leq t \leq T$$

$$P_2(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_2 t)$$

$$0 \leq t \leq T$$

T contains integer
cycles of $P_1(t)$

$$T = \frac{k_1}{f_1}$$

$$k_1 \neq k_2$$

$$T = \frac{k_2}{f_2}$$

T contains integer
cycles of $P_2(t)$

$$F_1 = \frac{K_1}{T} = K_1 F_0$$

$$F_2 = \frac{K_2}{T} = K_2 F_0$$

$$F_0 = \frac{1}{T_0}$$



Are different
multiples of same
fundamental freq
 F_0 .

$$E_1 = \int_{-\infty}^{\infty} P_1^2(t) dt$$

$$= \int_0^T \frac{2}{T} \cos^2(2\pi f t) dt$$

$$= \frac{2}{T} \int_0^T \frac{1 + \cos(4\pi f t)}{2} dt$$

$$= \frac{2}{T} \cdot \frac{T}{2} + \frac{1}{T} \cdot \frac{\sin 4\pi f t}{4\pi f} \Big|_0^T$$

$$= 1 + 0 = 1 = \epsilon_p.$$

Similary, $E_2 = \int_{-\infty}^{\infty} P_2^2(t) dt = 1$
 $= E_p.$

Consider now the inner product

$$\int_{-\infty}^{\infty} P_1(t) P_2(t) dt$$
$$= \frac{2}{T} \int_{-\infty}^{\infty} \cos(2\pi f_1 t) \cos(2\pi f_2 t) dt$$

$$= \frac{1}{T} \int_0^T \left\{ \cos(2\pi f_1 + f_2)t + \cos(2\pi(f_1 - f_2)t) \right\} dt$$

$$f_1 + f_2 = \frac{k_1}{T} + \frac{k_2}{T}$$

$$= (k_1 + k_2) F_0$$

$$f_1 - f_2 = (k_1 - k_2) F_0.$$

$$= \frac{1}{T} \cdot \frac{\sin 2\pi(f_1 + f_2)t}{2\pi(f_1 + f_2)} \Big|_0^T$$

$$+ \frac{1}{T} \cdot \frac{\sin 2\pi(f_1 - f_2)t}{2\pi(f_1 - f_2)} \Big|_0^T$$

$$= \frac{1}{T} \cdot \frac{\sin 2\pi(k_1 + k_2)f_0 t}{2\pi(k_1 + k_2)f_0} \Big|_0^T$$

$$+ \frac{1}{T} \frac{\sin 2\pi(k_1 - k_2)f_0 t}{2\pi(k_1 - k_2)f_0} \Big|_0^T$$

$$\sin 2\pi(k_1 + k_2)f_0 T$$

$$= \sin 2\pi(k_1 + k_2) = 0$$

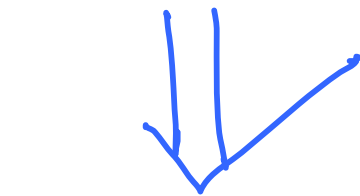
$$\begin{aligned} \sin 2\pi(k_1 - k_2) f_0 T \\ = \sin 2\pi(k_1 - k_2) \\ = 0. \end{aligned}$$

$$\int_{-\infty}^{\infty} P_1(t) P_2(t) dt = 0.$$

Inner product = 0

$\Rightarrow P_1(t), P_2(t)$ are orthogonal.

$p_1(t), p_2(t)$ Orthogonal
unit Energy



orthonormal

Basis for a signal space.

Signal space.

$$\begin{aligned} & \alpha_1 p_1(t) + \alpha_2 p_2(t) \\ &= \alpha_1 \cdot \sqrt{\frac{2}{T}} \cdot \cos(2\pi f_1 t) + \alpha_2 \cdot \sqrt{\frac{2}{T}} \cos(2\pi f_2 t) \end{aligned}$$